NAG Toolbox for MATLAB

s21da

1 Purpose

s21da returns the value of the general elliptic integral of the second kind $F\left(z,k',a,b\right)$ for a complex argument z, via the function name.

2 Syntax

```
[result, ifail] = s21da(z, akp, a, b)
```

3 Description

s21da evaluates an approximation to the general elliptic integral of the second kind $F\left(z,k',a,b\right)$ given by

$$F\left(z,k',a,b\right)=\int_0^z \frac{a+b\zeta^2}{\left(1+\zeta^2\right)\sqrt{\left(1+\zeta^2\right)\left(1+k'^2\zeta^2\right)}}d\zeta,$$

where a and b are real parameters, z is a complex argument whose real part is nonnegative and k' is a real parameter (the *complementary modulus*). The evaluation of F is based on the Gauss transformation. Further details, in particular for the conformal mapping provided by F, can be found in Bulirsch 1960.

Special values include

$$F\left(z,k',1,1\right)=\int_0^z \frac{d\zeta}{\sqrt{\left(1+\zeta^2\right)\left(1+k'2\zeta^2\right)}},$$

or $F_1\left(z,k'\right)$ (the *elliptic integral of the first kind*) and

$$F\left(z,k',1,k'^2\right)=\int_0^z \frac{\sqrt{1+k'^2\zeta^2}}{\left(1+\zeta^2\right)\sqrt{1+\zeta^2}}d\zeta,$$

or $F_2\left(z,k'\right)$ (the *elliptic integral of the second kind*). Note that the values of $F_1\left(z,k'\right)$ and $F_2\left(z,k'\right)$ are equal to $\tan^{-1}(z)$ in the trivial case $k'=1$.

s21da is derived from an Algol 60 procedure given by Bulirsch 1960. Constraints are placed on the values of z and k' in order to avoid the possibility of machine overflow.

4 References

Bulirsch R 1960 Numerical calculation of elliptic integrals and elliptic functions *Numer. Math.* **7** 76–90

5 Parameters

5.1 Compulsory Input Parameters

1: **z – complex scalar**

The argument z of the function.

Constraints:

$0.0 \leq \mathrm{Re}(\mathbf{z}) \leq \lambda$;
$\mathrm{abs}(\mathrm{Im}(\mathbf{z})) \leq \lambda$, where $\lambda^6 = 1/\mathrm{x02am}$.

2: **akp – double scalar**

The argument k' of the function.

Constraint: ABS(**akp**) $\leq \lambda$.

3: **a – double scalar**

The argument a of the function.

4: **b – double scalar**

The argument b of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – complex scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, Re(**z**) < 0.0,
or Re(**z**) $> \lambda$,
or ABS(Im(**z**)) $> \lambda$,
or ABS(**akp**) $> \lambda$, where $\lambda^6 = 1/\text{x02am}$.

ifail $= 2$

The iterative procedure used to evaluate the integral has failed to converge. The result is returned as zero.

7 Accuracy

In principle the function is capable of achieving full relative precision in the computed values. However, the accuracy obtainable in practice depends on the accuracy of the standard elementary functions such as ATAN2 and LOG.

8 Further Comments

None.

9 Example

```
z = complex(1.2, +3.7);
akp = 0.5;
a = 1;
b = 1;
[result, ifail] = s21da(z, akp, a, b)
```

```
result =
   1.9713 + 0.5054i
ifail =
           0
```